

# Colour Contrasts and Associated Metrics Defined in the LIP Framework. Generated Neighbourhoods in the (x, y)-space.

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## Introduction

The LIP (Logarithmic Image Processing) Model is now recognized as a powerful framework to process images acquired in transmitted light and to take into account the Human Visual System. In this paper, we first recall the concept of Logarithmic Additive Contrast (LAC) for grey level images and we extend it to colour images. From this colour logarithmic contrast, various metrics are derived which possess a strong stability in presence of lighting variations or lighting drift. Moreover, thanks to the consistency of the LIP Model with Human Vision, it seemed us interesting to select a set of colours in the (x, y)-space and then to define their neighbourhoods for logarithmic metrics. The opportunity to compare them with Mac-Adam's ellipses is discussed.

## 1- Introduction, notations and recalls

Introduced by Jourlin et al ([1], [2], [3]), the LIP (Logarithmic Image Processing) Model proposes first a physical and mathematical framework adapted to grey level images acquired in transmitted light (when the observed object is placed between the source and the sensor). Based on the transmittance law, the LIP Model proposes two operations allowing the addition of two images and the multiplication of an image by a real number, each of them resulting in a novel image. Such operations possess strong mathematical properties, as recalled hereafter. Furthermore, the demonstration by Brailean ([4]), of the LIP Model consistency with human vision, considerably enlarges the application field of the Model, particularly for images acquired in reflected light on which we aim at simulating human visual interpretation.

In the context of transmitted light, each grey level image may be identified to the observed object, as long as the acquisition conditions (source intensity and sensor aperture) remain stable. An image  $f$  is defined on a spatial support  $D$ , with values in the grey scale  $[0, M[$ , which may be written :

$$f: D \subset \mathbb{R}^2 \rightarrow [0, M[ \subset \mathbb{R}$$

Note that within the LIP Model, 0 corresponds to the « white » extremity of the grey scale, which means to the source intensity, i.e. when no obstacle (object) is placed between the source and the sensor. The reason of this grey scale inversion is justified by the fact that 0 will appear as the neutral element of the addition law defined in formula (3). The other extremity  $M$  is a limit situation where no element of the source is transmitted (black value). This value is excluded of the scale, and when working with 8-bits digitized images, the 256 grey levels correspond to the interval of integers  $[0:255]$ .

The transmittance  $T_f(x)$  of an image  $f$  at  $x \in D$  is defined by the ratio of the outcoming flux at  $x$  by the incoming flux (intensity of the source). In a mathematical formulation,  $T_f(x)$  may be understood as the probability, for a particle of the source incident at  $x$ , to pass through the obstacle, which means to be seen by the sensor.

The addition of two images  $f$  and  $g$  corresponds to the superposition of the obstacles (objects) generating respectively  $f$  and  $g$ . The resulting image will be noted  $f \triangle g$ . Such an addition is deduced from the transmittance law:

$$T_{f \triangle g} = T_f \cdot T_g \quad (1)$$

This formula means that the probability, for a particle emitted by the source, to pass through the “sum” of the obstacles  $f$  and  $g$ , equals the product of the probabilities to pass through  $f$  and  $g$ , respectively. The link between the transmittance  $T_f(x)$  and the grey level  $f(x)$  is given by ([3]):

$$T_f(x) = 1 - \frac{f(x)}{M} \quad (2)$$

Replacing in formula (1) the transmittances by their values obtained in (2) yields:

$$f \triangle g = f + g - \frac{f \cdot g}{M} \quad (3)$$

From this addition law, it is possible ([1], [3]) to derive the multiplication of an image by a real number  $\lambda$  according to:

$$\lambda \triangle f = M - M(1 - \frac{f}{M})^\lambda \quad (4)$$

Remark 1: such laws satisfy strong mathematical properties. In fact, if  $I(D, [0, M[)$  and  $F(D, ] - \infty, M])$  design respectively the set of images defined on  $D$  with values in  $[0, M[$ , and the set of functions defined on  $D$  with values in  $] - \infty, M[$ , we have ([3]):

$(F(D, ] - \infty, M]), \triangle, \triangle)$  is a real vector space and  $(I(D, [0, M]), \triangle, \triangle)$  is its positive cone.

Remark 2: if  $f(x, y) \geq g(x, y) \quad \forall (x, y) \in D$ , it is possible to define the subtraction  $f \triangle g$  according to:

$$f \triangle g = \frac{f - g}{1 - \frac{g}{M}}$$

## II- Logarithmic Additive Contrast for grey level and colour images

### 1- For grey level images

In the LIP framework, Jourlin et al. ([5]) introduced the Logarithmic Additive Contrast  $LAC_f(x, y)$  of a grey level function  $f$  at a pair  $(x, y)$  of points lying in  $D^2$  according to the formula :

$$\text{Min}(f(x), f(y)) \triangle (LAC_f(x, y)) = \text{Max}(f(x), f(y)) \quad (5)$$

“Optical” interpretation: Such a contrast represents the grey level which must be added (superposed) to the brightest point (smallest grey level) in order to obtain the darkest one (highest grey level), which implies that this logarithmic contrast may be visualized without normalization.

Using the addition formula (3) yields:

$$LAC_f(x, y) = \frac{|f(x) - f(y)|}{1 - \text{Min}(f(x), f(y))/M} \quad (6)$$

### 2- Extension of LAC to colour images and associated metrics

Considering two colours  $C_1 = (R_1, G_1, B_1)$  and  $C_2 = (R_2, G_2, B_2)$  in the RGB-space, we propose to introduce the Logarithmic Additive Contrast in each channel. For example in the red channel,  $LAC_R(C_1, C_2)$ , or  $LAC_R$  when no confusion is possible, is defined according to:

$$LAC_R(C1, C2) = \text{Max}(R1, R2) \triangle \text{Min}(R1, R2) = \frac{|R1 - R2|}{1 - \text{Min}(R1, R2)/M}$$

From the three contrasts  $LAC_R$ ,  $LAC_G$  and  $LAC_B$ , we can deduct various metrics, for example:

$$d1 = \frac{1}{\sqrt{3}} \cdot \sqrt{LAC_R^2 + LAC_G^2 + LAC_B^2} \quad \text{and} \quad d2 = \frac{1}{3} \triangle (LAC_R \triangle LAC_G \triangle LAC_B)$$

Another approach (cf. Fig. 1) consists of using a “probing” technique, as proposed by Asplünd ([6], [7]) to evaluate the distance between two binary shapes A and B according to:

$$d_{As}(A, B) = \ln\left(\frac{\lambda}{\mu}\right) \quad \text{where } \lambda = \text{Inf} \{ \lambda, A \subseteq \lambda B \} \text{ and } \mu = \text{Sup} \{ \mu, \mu B \subseteq A \}$$

The main property of such a distance is its independence to the magnifications of A and B.

This metric has been extended to grey level functions (cf. Fig. 2), using a “Double Sided Probing” approach ([8], [9]):

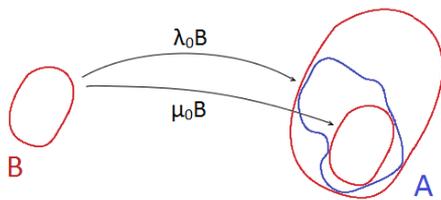


Figure 1- Asplünd's distance for binary shape

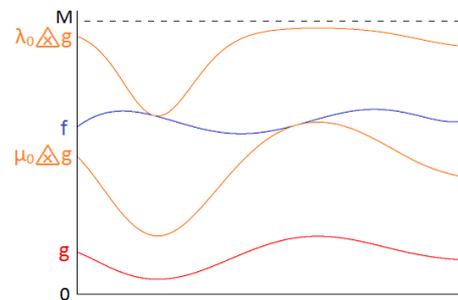


Figure 2- Asplünd's distance between two grey level functions

What we propose now is a similar approach to compute the “Asplünd's distance”  $d_{As}(C1, C2)$  between two colours  $C1 = (R1, G1, B1)$  and  $C2 = (R2, G2, B2)$

$$d_{As}(C1, C2) = \ln\left(\frac{\lambda}{\mu}\right) \quad \text{where } \lambda = \text{Inf} \{ \lambda, \lambda \triangle R1 \geq R2, \lambda \triangle G1 \geq G2, \lambda \triangle B1 \geq B2 \}$$

$$\text{and } \mu = \text{Sup} \{ \mu, \mu \triangle R1 \leq R2, \mu \triangle G1 \leq G2, \mu \triangle B1 \leq B2 \}$$

Fundamental remark: the main interest of these colour metrics arise from the consistency of the LIP framework with Human Vision. This point let expect a link between the shapes of colour neighbourhoods computed for each metric in the (x,y)-space and the elliptic shapes obtained by Mac Adam ([10]). This is the object of the next section.

### III- Logarithmic Metrics and associated neighbourhoods in (x,y) space

The Mac Adam experiment consisted of defining colours neighbourhoods for the human visual system. A neighbourhood of a colour C groups all the colours which are confused with C by an average person. For this purpose, Mac Adam asked people to identify all the colours looking like C. Then he represented the neighbourhood of each colour from a set of about 100, which appeared to be ellipses.

The consistency of the LIP framework with the human visual system is not formally established for colour images. That's why we decided to display the neighbourhoods associated to ours metrics in the (x,y) space, hoping some similarity with Mac-Adam ellipses. The difficulty is that we can't create constant luminosity colours as the Mac Adam ones because of our RGB representation.

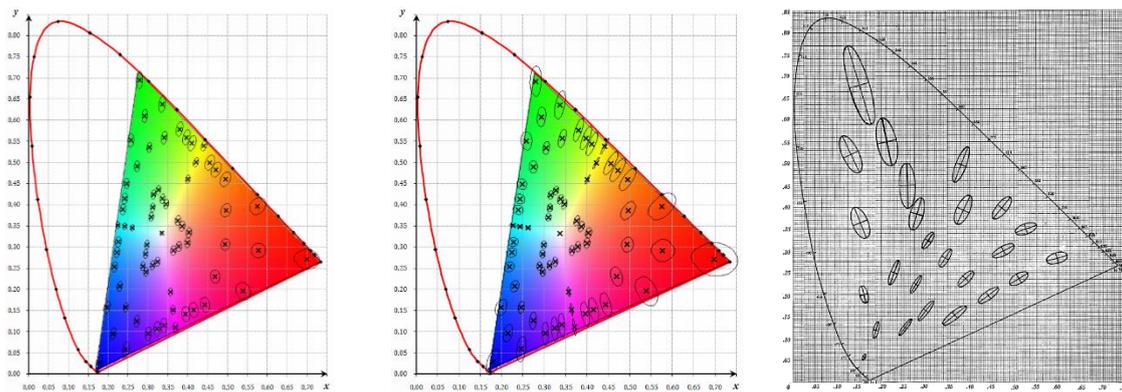


Figure 2- Euclidian colour neighbourhoods,  $d_1$ - colour neighbourhoods and MacAdam neighbourhoods

The neighbourhoods are limited to RGB colours as our metrics are based on the LIP model. Euclidian metric leads to circular shaped neighbourhoods, with a little distortion in the monochromatic parts of the diagram (Fig. 2, on the left). However, LIP-based neighbourhoods are elliptically shaped with a small size in the blue part and a larger one in the red and green ones (Fig. 2, middle) as in the Mac Adam experiment (Fig. 2, on the right).

### Conclusion and perspectives

In the present paper, we proved the ability of Logarithmic contrasts to generate Colour Metrics. Such Colour Metrics take into account the human eye sensitivity. A first step has been established in link with Mac-Adam ellipses. This point will be studied in depth in a near future, as well as the observed insensitivity of logarithmic concepts (contrasts, metrics) to lighting variations and/or lighting drift.

### References

- [1] Jourlin M., Pinoli J.C., 1988, "A model for logarithmic image processing", *J. Microsc.*, 149, 21-35
- [2] Jourlin M., Pinoli J.C., 1995, "Image dynamic range enhancement and stabilization in the context of the logarithmic image processing model", *Signal Process.*, 41 (2), 225-237
- [3] Jourlin M., Pinoli J.C., 2001, "Logarithmic Image Processing: The mathematical and physical framework for the representation and processing of transmitted images", *Advances in imaging and electron physics*, 115, 129-196
- [4] Brailean J., Sullivan B., Chen C., Giger M., 1991, "Evaluating the em algorithm using a human visual fidelity criterion", *Int. Conf. on Acoustics. Speech and Signal Processing*, 2957-2960
- [5] Jourlin M., Pinoli J.C., Zeboudj R., 1989, "Contrast definition and contour detection for logarithmic images", *J. Microsc.*, 156, 33-40
- [6] Asplünd E., 1960, "Comparison between plane symmetric convex bodies and parallelograms", *Mathematica Scandinavica*, vol. 8, 171-180
- [7] Grunbaum B., 1963, "Measures of symmetry for convex sets", *Symposia in Pure Mathematics*, vol. 7, 233-270
- [8] Barat C., Ducottet C. and Jourlin M., 2010, "Virtual double-sided image probing: A unifying framework for non-linear grayscale pattern matching", *Pattern Recogn.*, vol. 43, no. 10, pp. 3433-3447
- [9] Jourlin M., Carré M., Breugnot J., Bouabdellah M., 2012, "Logarithmic Image Processing: Additive Contrast, Multiplicative Contrast, and Associated Metrics", *Advances in Imaging and Electron Physics*, Vol. 171, 357-406.
- [10] Mac Adam, D. L., 1942, "Visual sensitivities to colour differences in daylight", *J. Opt. Soc. Am.*, 32, 247-274