A CLASS OF STOCHASTIC MODELS FOR 'NORMAL' SPATIAL POLYHEDRAL TESSELLATIONS, IN WHICH 3 FACES MEET AT EACH EDGE, AND 4 AT EACH VERTEX

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The simplest stochastic model for such a tessellation in space ($\mathbb{R}^3$) is the Voronoi, V(3), based on a Poisson process base in $\mathbb{R}^3$. That is, each Poisson 'particle' is the 'nucleus' of the polyhedral set of points of $\mathbb{R}^3$ closer to it than to any other particle. The mean number of plane faces for the polyhedra of V(3) is (Meijering, 1953)

$$E_3(N) = 2 + (48\pi^2/35) = 15.54 \ .$$

For fitting purposes, it is desirable to have a class of such random tessellations, with varying E(N) value. Perhaps the simplest such class is that of the Sectional Voronoi tessellations

$$V(3,d) = [\text{standard Voronoi tessellation } V(d) \text{ in } \mathbb{R}^d]$$

$$\cap \ [\text{3-flat in } \mathbb{R}^d] \qquad (d = 3,4,\ldots).$$

Thus $V(3,3) = V(3)$. As d increases from 3 to ∞ , the corresponding mean $E_{3,d}(N)$ ranges (Miles, 1984) from 15.54 down to

$$2 + \{(1/4) - (3/2\pi)\sin^{-1}(1/3)\}^{-1} = 13.40 \ .$$

Stereological aspects are discussed.

REFERENCES

Meijering, J.L. Interface area, edge length, and number of vertices in crystal aggregates with random nucleation. Philips Res.Rep.,1953; 8: 270-290.

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