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A Real Analytic Schwartz' Kernels Theorem

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Abstract

In this short paper, we study a few topological properties of the sheaf of real analytic functions on a real analytic manifold M. In particular, we show that its topological Poincaré-Verdier dual is the sheaf of hyperfunction densities on M. We also prove that if N is a second real analytic manifold, then the continuous cohomological correspondences between the sheaf of real analytic functions on M and the sheaf of hyperfunctions on N are given by integral transforms whose kernels are hyperfunction forms on $M \times N$ of a suitable kind. This result may be viewed as a real analytic analogue of the well-known kernels theorem of Schwartz.

Introduction

Let M be a real analytic manifold of dimension m and let X be a complexification of M. Denote or_M the orientation sheaf of M and $\mathcal{O}_X$ the sheaf of holomorphic functions on X. A classical pure codimensionality theorem due to M. Sato sates that all the cohomology sheaves of the complex

$$\mathrm{or}_M \otimes \mathrm{R}\Gamma_M(\mathcal{O}_X)$$

vanish except for the m-th one. This non-vanishing cohomology sheaf is then defined to be the sheaf $\mathcal{B}_M$ of hyperfunctions on M.

This approach is at first glance completely different from the one followed by L. Schwartz to construct the sheaf of distributions on a smooth manifold. Recall that, if M is a smooth manifold of dimension m, one defines the sheaf $\mathcal{D}b$ of distributions on M by duality through the formula

$$\mathcal{D}b(U) = \mathrm{L}\left(\Gamma_c(U; \mathrm{or}_M \otimes \mathcal{C}^m_\infty), \mathbb{C}\right)$$

where $\mathcal{C}^m_\infty)$ is the sheaf of smooth m-forms on M and U is any open subset of M.