

Eclipse Timing Variations Observed in PCEB Systems

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Abstract

Post-common Envelope Eclipsing Binary (PCEB) systems with a white or sub-dwarf primary and a low-mass secondary are ideal systems to study Eclipse Timing Variations (ETVs) due to their relatively small total masses which make it easy to detect gravitationally bound, yet unseen, additional bodies even within planetary limits on long-period orbits. With the addition of the recent discovery of two planetary-mass bodies in the sdB + dM binary Kepler-451 system, the number of circumbinary planets in PCEBs approaches 30 now. However, some of these systems turned out to be dynamically unstable with the suggested planetary parameters. As more observations accumulate over time, there have been significant changes in both the number of suggested planets and their parameters. These necessitate a look back to the ETV analyses of these systems with new and more precise data spanning longer baselines. Within this contribution, we review the systems hosting substellar bodies suggested from the light-time effect they are claimed to be causing. We list the general problems encountered during the ETV analyses and the interpretation of the results, and comment on their possible solutions. We calculate and present the potentials of supporting evidence from astrometry, radial velocity, and transit observations as well as tests through dynamical stability analyses and magnetic activity assumptions.

Keywords: close binary stars, eclipse timing, O-C analysis, photometry

1. Introduction

In a close binary system, following the main-sequence evolution of the more massive component, matter from its outer layers starts to be transferred to the less massive companion. This companion cannot accrete all this matter, which forms a common envelope around the binary system. The stars then transfer angular momentum to this common envelope material due to friction, which causes them move towards each other while the common envelope is ejected with the gained momentum in a very short period of time. Such systems end up with an O- or B-type hot subdwarf (sdOB) or white dwarf (WD) primary depending on the initial mass and evolutionary history of the more massive star, and a low mass secondary, orbiting about the common center of mass within a couple of hours. These systems are called Post Common Envelope Binary (PCEB) systems (Sarna et al., 1995).

Timing variations in the eclipses observed in the PCEB systems with a favourable orbital inclination are interesting in particular. Almost all eclipsing PCEBs with sdOB primaries and dwarf M (dM) secondaries display Eclipse Timing Variations (ETVs) (see the references in the footnote of Table 1) with the exception of AA Doradus (Baran et al., 2021). They are especially helpful in determining the eclipse times within very high precision thanks to their V-shaped eclipses and short orbital periods (Parsons et al., 2010). With the help of precise photometry with continuous coverage and long baselines, it has become possible to study variations at longer timescales and lower amplitudes especially from space-based observatories. These observational advantages facilitate potential periodic variations due to gravitationally bound, yet unseen, additional bodies to be revealed, which would cause low-amplitude, long-period ETVs. Out of 31 circumbinary exoplanets discovered with the eclipse timing technique, 29 of these have been found around PCEBs according to the NASA Exoplanet Archive (<https://exoplanetarchive.ipac.caltech.edu/>) and the Exoplanets Encyclopedia (<http://exoplanet.eu>).

PCEB systems are also excellent laboratories to study planet formation and evolution in environments other than single solar-like stars. The ejected common envelope material can form a circumbinary disk, in which low-mass bodies, such as planets or brown dwarfs might form through either core accretion or disk instability. Such bodies are referred as "second generation planets". On the other hand, there are theoretical explanations predicting survival scenarios for the so-called "first generation planets". If these substellar bodies had formed simultaneously with the binary system, then at least their orbital parameters should have changed within time. It is possible that they also accrete some more mass from the ejected material by the evolving system that we observe as a PCEB now and become more massive within a scenario deemed as "hybrid" (Zorotovic and Schreiber, 2013). Finding these objects and constraining these parameters will help us understand the chemical composition of the ejecta as well, which should be hydrogen / helium-rich if it is not enriched by some other mechanism. This can shed some light upon the problems related to mixing in hot subdwarf atmospheres.

However, there are several issues which require attention in the ETV analysis, which can prevent us from revealing small amplitude variations over long timescales and / or cause misinterpreting its results. First of all, the fundamental data sets for ETV analyses, eclipse timings, originate from a variety of sources and techniques utilizing different sets of software packages and algorithms. This leads to significant heteroscedasticity in the data, which must be handled carefully and appropriately. In addition, we have insufficient knowledge about the noise levels indicated by error bars in the timing measurements, which are known to be underestimated by some techniques frequently employed for measurements (Kwee and van Woerden, 1956). The noise in the data always brings orbital eccentricity in the modelling because it has a one-tailed distribution with a maximum at zero (circular orbit). The degeneracies in the models are further aggravated by the parameters of the additional causes of ETVs, such as magnetic activity and / or angular momentum loss / transfer. We only have a naive understanding of activity-induced quadruple moment changes, and since there should be no mass transfer between the components of a PCEB system, secular changes, which have been employed quite extensively in the literature, have little physical basis. Even though the orbital periods are short, gravitational

wave emission can only be responsible for a small fraction of detected ETV signals since the masses are small. Moreover, stellar winds should be very weak given the large surface gravities of the components and should also not contribute much. Confusing periodic changes with the secular is perfectly possible since the latter can be occurring at a much longer timescale than the baseline of observations.

All these issues lead to different explanations of the ETV variations observed in a system as more and better precision data sets are acquired. On the other hand, additional bodies assumed to be gravitationally bound to explain periodic ETV variations were also found to be unstable in a significant number of PCEBs. We discuss these problems encountered in ETV analysis and suggest potential solutions to a few of them based on a study of all well-known and new PCEB systems around which substellar-mass bodies have been suggested with the ETV technique within this contribution.

2. Potential of Independent Methods for Confirmation

There are currently 29 companions within the mass limits of planets and brown dwarfs, orbiting 20 eclipsing PCEB systems discovered thanks to the variations they induce on eclipse timings according to the NASA Exoplanet Archive and the Extrasolar Planet Encyclopedia. All of these substellar objects have been suggested through the observations of the light-time effect (LiTE) that they are claimed to be causing. The orbits of some of these bodies have been found to be unstable with the suggested parameters, while there could be alternative explanations of the observed ETVs for some others such as magnetic activity-induced quadruple moment changes (e.g. Applegate, 1992; Lanza, 2020). Apical motion, on the other hand, should not be expected from such binaries with short-period, circular orbits. Therefore, independent methods are badly needed to reject the null hypothesis.

We list the properties of these planets in Table 1 that we obtained from the literature we provided in the footnotes of the table, based on which we derive the total amplitudes of the corresponding radial velocity and direct imaging signals (maximum angular distance) assuming coplanar orbits with the eclipsing binary ($i_3 \sim 90^\circ$) from numerical integrations of the systems for a time of a few orbital revolution of the outermost object. We performed these numerical integrations by using the Rebound (Rein and Liu, 2012) package. The calculations of the amplitudes were made as explained in Fabrycky (2011). We also calculated the a-priori transit probabilities based on the geometric transit probability expressed as

$$p_{tr} = \frac{R_\star + R_p}{a} \frac{1 + e \sin \omega}{1 - e^2} \quad (1)$$

We obtained the stellar radii (R_\star) from the same literature either based on derived radii values from light and / or radial velocity curve analyses, or canonical values for the WD or sdOB primaries, which dominate the total light. Since the flux contribution of the companion is next to none, we computed the probabilities only for the transits in front of the primary stars. Planetary radii (R_p) are not known because any of their transits has not been observed so far. Hence, we employed the mass-radius relation provided by Chen and Kipping (2017) assuming

Table 1: Amplitudes of radial velocity (A_{RV}) & imaging (A_{imaging}) signals, and a-priori transit probabilities ($p_{\text{tr,ecc.}}$) expected from ETV systems based on planet parameters and best ETV models from the literature and assumptions of a circular orbit ($p_{\text{tr,circ.}}$).

Planet	Mass (M_{Jup})	Radius (R_{Jup})	a (AU)	e	ω ($^{\circ}$)	R_{\star} (R_{\odot})	P_{ETV} (years)	A_{ETV} (s)	A_{RV} (m/s)	A_{imaging} (mas)	$P_{\text{tr,ecc.}}$ (%)	$P_{\text{tr,circ.}}$ (%)
DP Leo b ¹	6.05	1.33	8.19	0.39	-78.00	0.0115	28.80	66.7	155.4	47.29	0.0005	0.0007
KIC 10544976 b ²	13.40	1.43	6.56	0.29	211.00	0.0136	16.80	80.10	308.9	24.78	0.0009	0.0010
OY Car b ³	8.48	1.08	6.18	0	-	0.0113	14.00	65.4	222.1	135.00	-	0.00008
RR Cae b ⁴	4.20	0.98	5.30	0	-	0.0155	11.90	33.9	131.1	497.13	-	0.0014
V2051 Oph b ⁵	7.30	1.37	9.00	0.37	190.20	0.0103	21.64	62.2	153.9	8.94	0.0006	0.0005
DE CVn b ⁶	11.50	0.87	5.75	0	-	0.0136	11.22	66.9	281.1	372.49	-	0.0011
QS Vir ⁷	6.44	0.94	4.20	0.37	38.30	0.0107	7.86	21.1	177.8	161.97	0.0017	0.0012
V893 Sco ⁸	9.50	1.22	4.50	0.30	-	0.0100	10.19	43.7	283.1	72.08	-	0.0010
DV UMa b ⁹	26.20	0.92	8.60	0.44	26.11	0.0070	17.58	149.5	492.6	42.81	0.0006	0.0004
DD CrB b ¹⁰	1.36	1.25	-	0	-	0.1720	10.46	9.0	51.6	15.05	-	0.0010
SDSS-1456 b ¹¹	16.70	0.79	-	0.05	23.70	0.0145	13.05	109.0	500.0	53.01	-	0.0008
GK Vir b ¹²	0.95	0.99	7.38	0.14	198.0	0.0170	24.34	9.7	24.3	29.87	0.0010	0.0011
HW Vir b ¹³	25.10	1.02	7.90	0.45	359.0	-	-	-	-	-	0.0128	0.0103
HW Vir c ¹³	13.90	1.11	4.57	0.27	13.0	0.1750	28.21	317.0	1398.7	88.18	0.0204	0.0178
Kepler-451 b ¹⁴	1.86	1.14	0.90	0.33	302.0	-	-	-	-	-	0.0856	0.1059
Kepler-451 c ¹⁴	1.61	1.49	2.10	0.29	7.0	-	-	-	-	-	0.0513	0.0454
Kepler-451 d ¹⁴	1.76	1.31	0.20	0	-	0.0205	4.93	8.9	512.8	11.75	-	0.4767
NN Ser b ¹⁵	7.33	1.57	5.35	0.08	43.0	-	-	-	-	-	0.0019	0.0018
NN Ser c ¹⁵	2.30	0.98	3.43	0.19	249.0	0.0211	16.05	67.4	305.6	21.27	0.0024	0.0029
V1828 Aql b ¹⁶	8.00	1.05	2.90	0.52	98.0	-	-	-	-	-	0.0626	0.0301
V1828 Aql c ¹⁶	2.90	0.88	1.90	0	-	0.1880	7.00	50.2	534.8	7.85	-	0.0460
NY Vir b ¹⁷	2.78	1.11	3.39	0	-	-	-	-	-	-	-	0.0206
NY Vir c ¹⁷	4.49	1.05	7.54	0.44	333.0	0.1500	27.00	56.8	238.2	24.20	0.0092	0.0093
UZ For b ¹⁸	10.00	1.08	5.70	0.69	120.3	-	-	-	-	-	0.0029	0.0009
UZ For c ¹⁸	3.22	0.98	3.00	0.45	347.4	0.0113	14.75	73.5	367.6	47.14	0.0020	0.0017
V470 Cam b ¹⁹	28.30	1.24	3.27	0	-	-	-	-	-	-	-	0.0327
V470 Cam c ¹⁹	12.40	1.35	4.71	0	-	0.2300	14.48	171.0	1672.2	7.98	-	0.0227
HU Aqr b ²⁰	16.80	0.95	5.48	0.23	92.3	-	-	-	-	-	0.0009	0.0007
HU Aqr c ²⁰	20.80	0.86	6.38	0.08	72.6	0.0080	19.51	181.0	857.4	77.4	0.0006	0.0006

¹Beuermann et al. (2011), ²Almeida et al. (2019), ³Han et al. (2015), ⁴Qian et al. (2012), ⁵Qian et al. (2015), ⁶Han et al. (2018), ⁷Qian et al. (2010), ⁸Bruch (2014), ⁹Han et al. (2017), ^{10,11}Wolf et al. (2021), ¹²Almeida et al. (2020), ¹³Esmer et al. (2021), ¹⁴Esmer et al. (2022), ¹⁵Marsh et al. (2014), ¹⁶Almeida et al. (2013), ¹⁷Lee et al. (2014), ¹⁸Khangale et al. (2019), ¹⁹Sale et al. (2020), ²⁰Goździewski et al. (2015)

the planetary orbit is coplanar with the eclipsing binary. We used the eccentricity (e) value and the argument of periastron (ω) when they are provided in the relevant literature. We repeated the calculations assuming circular orbits ($e = 0$) for all the planets.

3. Magnetic Activity Assumption

In order to test the magnetic activity assumptions as an alternative explanation of the observed ETVs in the systems listed in Table 1, we made use of the online tool of Völschow et al. (2016) (<http://theory-starformation-group.cl/applegate/index.php>) that makes it possible to calculate the energy ($\Delta E / E_{\text{sec}}$) induced by the magnetic activity attributed to the secondary

Table 2: Ratios of the observed ETV amplitude and the amplitude expected from a quadruple moment change that can be induced by the magnetic activity of the secondary star with the assumed periods of 5, 10 and 20 years based on two-shelled model by Völschow et al. (2016).

System	P_{ETV} (years)	A_{ETV} (s)	$\Delta E / E_{\text{sec}}$	$A_{\text{quad},5\text{yrs}}$ (s)	$A_{\text{quad},10\text{yrs}}$ (s)	$A_{\text{quad},20\text{yrs}}$ (s)	A_{q5} / A_{ETV}	A_{q10} / A_{ETV}	A_{q20} / A_{ETV}
DP Leo	28.80	66.7	1.40	4.09	11.56	32.69	0.06	0.17	0.49
KIC 10544976	16.80	80.1	70.50	1.67	4.72	13.32	0.02	0.06	0.17
RR Cae	11.90	33.9	447.20	4.30	1.53	0.54	0.13	0.05	0.02
QS Vi	7.86	21.1	0.79	12.22	34.53	97.54	0.58	1.64	4.62
DD CrB	10.46	9.0	8.01	1.06	3.00	8.48	0.12	0.33	0.94
HW Vir	28.21	317.0	26.55	4.85	13.71	38.67	0.02	0.04	0.12
Kepler-451	4.93	8.9	19.85	2.06	5.83	16.45	0.23	0.65	1.85
NN Ser	16.05	67.4	85.19	1.33	3.76	10.62	0.02	0.06	0.16
V1828 Aql	7.00	50.2	109.39	2.96	8.39	23.69	0.06	0.17	0.47
NY Vir	27.00	56.8	2.45	2.91	8.21	23.30	0.05	0.14	0.41
UZ For	14.75	73.5	10.67	4.48	12.65	35.76	0.06	0.17	0.49
V470 Cam	14.48	171.0	43.31	5.40	15.20	42.99	0.03	0.09	0.25
HU Aqr	19.61	181.0	4.23	11.41	35.25	91.11	0.06	0.18	0.50

stars in these systems. For this purpose, we used two-shelled model of Völschow et al. (2016). Based on these energies, we calculated the corresponding ETV amplitudes assuming 5, 10, and 20 years of activity cycles, compared them with the actually observed ETV amplitudes, and provided them together with these ratios in Table 2. The period values were selected as such because they are comparable to the time-scales of magnetic activity cycles in cold stars of different ages. QS Virginis was found to be the only system for which the observed ETV amplitude can be fully associated to the assumed magnetic activity of the M-type secondary. In other systems, only a fraction of the observed amplitude can be explained by magnetic activity induced quadruple moment changes according to the two-shelled model by Völschow et al. (2016).

We would like to note that determination of the temperature and the luminosity of the secondaries in PCEB systems is not a trivial task due to insufficient amount of signal from the secondary companion during both photometric and spectroscopic observations. Our calculations for the magnetic energy of the secondary depend strongly on the values of these parameters that we adopted from the literature. Hence, the uncertainties of $\Delta E / E_{\text{sec}}$ may be large enough to prevent a robust interpretation. Since we do not possess detailed knowledge of these uncertainties, we interpret our results based on the published values directly.

4. Orbital Stabilities

Although orbital stability tests have been employed to interrogate the existence of potential substellar bodies, they are based on the best fits of the ETV data, most of which lead to highly-eccentric models. In addition, they are found to be superimposed on secular changes modeled with quadratic functions, the physical reasons for which are not well suited to the physics of PCEB systems. These secular changes can well be segments of longer-period variations, induced by other potentially additional bodies which can make a system stable. Moreover, assumptions of zero eccentricity will change the parameters of the underlying model, which can also make it stable.

Therefore, we performed stability tests for the systems with multiple planets (NN Ser, V1828 Aql, NY Vir, UZ For, V470 Cam and HU Aqr). For this purpose, we used Frequency Map Analysis (FMA) of Laskar (1990) and Laskar (1993). For the numerical integrations of orbits, we made use of the Rebound package. To find the fundamental frequencies representing the mean motion of the planets, we used TRIP package (Gastineau and Laskar, 2011). We fixed the eccentricities of the planets to zero and sampled the semi-major axes from various ranges to find the stable regions, which correspond to the logarithm of normalized stability index value of -6 (Correia et al., 2005). We adopted the best-fit solutions from the literature. Only for NY Vir, we used our own preliminary analysis from our upcoming study. We set timesteps of 25, 50 and 100 days depending on the orbital period of the inner planet, and set the total orbital integration time as 10^6 or 10^7 days depending on the orbital period of the outermost planet.

The stability tests for the multiplanet systems resulted in unstable orbital configurations within the parameter uncertainties, except for NY Vir. We note that the eccentricity values shown in Table 1 are very high for many systems, which can be the main cause of the orbital instability. Therefore, we prepared low resolution stability maps for the systems with the remaining multiple planets assuming zero eccentricities for their orbits.

We found potentially stable solutions within the uncertainties of the best fit models for the case of UZ For, NN Ser and V1828 Aql, but not for HU Aqr and V470 Cam. For the latter group, the assumption of circular orbits is insufficient to have stable configurations, and the separation of the planets is very likely to be responsible for that matter. An example of the effect of the circular orbit assumption is provided in Figure 1 for the WD + dM PCEB system NN Ser. Although Marsh et al. (2014) found the system to be unstable with the parameters of two planetary-mass bodies suggested as a result of the ETV analysis by Beuermann et al. (2010), when the eccentricities are fixed to zero with all other ETV parameters kept constant, the resultant system is found to be stable.

Another example of replacing the best ETV solution with a near-by potential solution in the parameter space is given by Esmer et al. (2022), who announce the discoveries of two Jovian-mass planets in the Kepler-451 system. Although the best ETV solution gave an orbital period of 1460 days for the outermost planet (c) which makes it unstable, a similar-quality fit with a very close χ^2_{ν} value ends up in an orbital period of ~ 1800 days for the planet-c, which makes it stable.

5. Discussion

Ever since the first gravitationally bound substellar-mass objects were suggested from ETV analyses, they have been a matter of debate since (i) 29 of 31 known ETV planets were found out of only 168 PCEB systems, and (ii) most PCEB systems with precise timing data over long observation baselines display ETVs. However, as the baseline of observations extended and it became possible to observe over the Nyquist frequency with sufficiently precise data, the leading position of LiTE amongst other potential explanations is being corroborated. Even current models of magnetic activity-induced quadruple moment changes (Völschow et al., 2016;

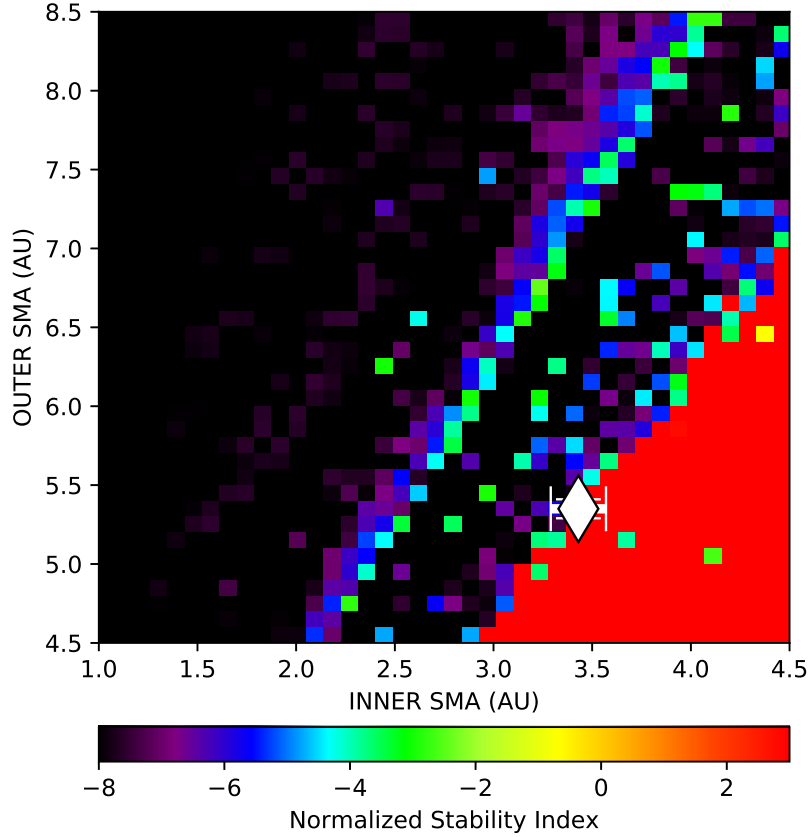


Figure 1: Stability map for the WD + dM system NN Ser based on the best ETV solution parameters (diamond and the error bars) other than the eccentricity, which is fixed to zero to produce this stability map based on the normalized stability index. The stable regions correspond to the log normalized stability index value of -6 or less (blue-purple-black regions).

Lanza, 2020) seem to account for only a small percentage of the observed ETVs.

In addition, as more data with better precision is being provided by especially space-borne observatories, smaller amplitude variations are being revealed which can be decisive on the orbital stability of the entire system, otherwise found to be unstable with fewer bodies having slightly different parameters derived from the analysis of ETV data with inferior quality. Kepler-451 (Esmer et al., 2022) is a sound example of this with the two shortest-period planets ever found within 43 and 406 day-orbits, respectively. Together with the discovery of the outer planet, Kepler-451 c, this system is found to be stable from almost an equally-probable ETV solution as that suggested by the best-fit model giving very similar fit statistics.

Nevertheless, we need independent methods to confirm the presence of these systems. Although the predicted RV amplitudes are well above the detection limits achieved for single stars, they are formidable to be observed for short-period eclipsing binaries with a very hot primary and a cool secondary because only detectable hydrogen and helium lines of the primary are significantly broadened in these essentially single-lined systems. Considering the low proba-

bilities of transits of planets with very large semi-major axes across very small components of PCEB systems, the only viable options seems to be direct imaging. Nevertheless, the only direct imaging observation attempt of such a system, V471 Tau, (Hardy et al., 2015) ended in a null detection. However, with better quality data spanning even longer baselines, it will be possible to have a larger sample of eclipsing PCEBs with observed ETVs, which will allow us to work with a diverse set of parameters. Then, it might be possible to observe directly a substellar object or a transit of it, which will constitute the first confirmation of an ETV planet.

Finally, we investigated the detectability of ETV planets with Gaia astrometry. For this purpose, we calculated the astrometric signature (α) values with the formulation given by Perryman et al. (2014). We used the minimum mass and semi-major axes of the planets, and for multiplanet system we used only the outermost planet's properties. We compared the astrometric signature values with the along-scan accuracy per field of view crossing (σ_{fov}) values given by Perryman et al. (2014) corresponding to their G brightnesses. Assuming the detection limit as $\alpha \geq \sigma_{fov}$, 14 out of 22 systems becomes detectable. If the detection limit becomes $\alpha \geq 3\sigma_{fov}$, 10 of the systems can be expected as detectable with Gaia, if sufficient number of observations are available that can cover a significant time of their long orbital periods. That said, the fraction of the orbital motion observed by Gaia can be confused with the proper motion of the center of mass of the binary, which should be taken into account considering the long orbital periods of ETV-planets around PCEBs.

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Conflicts of interest

The authors declare no conflict of interest.

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