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AN ADAPTIVE DISCRETE CONTOUR SMOOTHING

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ABSTRACT

This paper presents an original smoothing method for discrete contours containing different characteristics depending on the scale. The optimal scale is determined by a survey of the contours in terms of texture. An abstract can be then derived ; it consists in the inflexion and dominant points of the discrete contour under study and in the directions of the tangents at these points. The smoothing computed starting from such abstract uses the theory of straight line envelopes. Neither threshold nor experimental parameter are required and the smoothing provides the underlying continuous feature of a discrete contour seen at a given scale.

Keywords : discrete contour, edge texture indicator, scale, smoothing, straight line envelope.

INTRODUCTION

Edge detection is an important first step for many vision systems. Unfortunately, in most cases, contours derived from a scene are made up of connected points ; no geometric information is available. Furthermore such contours are noised essentially because of the limited resolution of rasters and sensors used when acquering the images. In addition, digital contours can present different characteristics depending on the scale (Bengtsson *et al.*, 1991 ; Moktarian *et al.*, 1986).

A smoothing approaching the underlying continuous curve which generated the discrete contour under study (Rubio, 1990) can be used to deal with these problems. However many smoothing methods require control points to describe the global shape of the resultant curve. These control points are not given by the discrete contours and the use of such smoothing methods is not possible without operator intervention. That is for example the case for smoothings based on Bézier curves or on splines.

Consequently we present a smoothing totally adapted to the available data that the points of a discrete contour are. This smoothing uses the theory of the envelopes of a one-parameter family of straight lines. Neither control point nor experimental threshold are required thus eliminating the unrewarding and questionnable parameter setting.

First we shall mention the problem of the scale of study. Then we shall describe the construction of a discrete contour's abstract before presenting the smoothing method itself. We shall end this paper by giving some examples illustrating the features of the method.

SCALE OF STUDY

Various information can be available on a discrete contour : the number of convex and concave parts is certainly the most easily readable. But, in general case, the contour C of a digitalized convex object K is a non convex polygon at pixel level (Fig. 1). Nevertheless it can be seen globally convex at a greater scale. Thus, to derive the more significant features conveyed by the contour of the original continuous shape it is necessary to choose an appropriate scale.

Let K be the result of the digitalization of a continuous object X. Its contour C can be denoted by a sequence of N integer-coordinate points (Freeman, 1974) :

$$C = \{p_i = (x_i, y_i), i = 1, ..., N\}$$

where p_{i+1} is a neighbor of p_i (modulus N). Unless otherwise stated, in the sequel, all integers are of modulus N. Let n be an integer such that $0 < n < N/2$. Note α_i the angle formed by the half-lines derived from a point p_i and passing respectively through p_{i-n} and p_{i+n} (Fig. 1). Set finally $A_n(i) = \pi - \alpha_i$.

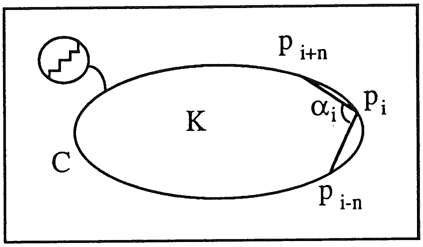


Fig. 1. Result of the digitalization of a continuous convex object.

Generally, when i varies, A_n has sign variations. These sign variations are due to the local convexity of the boundary of X. This remark leads to define a mapping $T_K : N^* \rightarrow N$ such that $T_K(n)$ is the number of sign changes of A_n ; n corresponding to the scale. This mapping has been introduced by (Rubio, 1990) and it is called *edge texture indicator.*

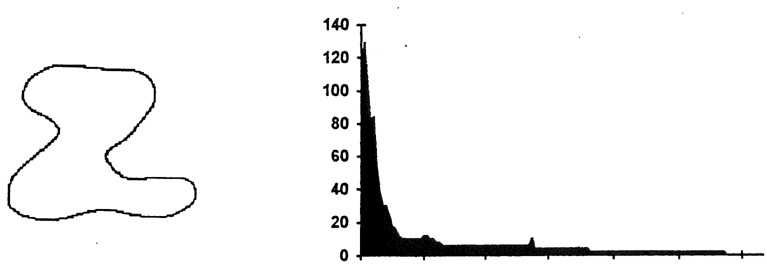


Fig. 2. The contour consists of 432 points and it contains some noise. The number of sign changes of A_n versus scale is shown in the graph. The plateaus of the edge texture indicator are easily seen.

If the boundary of X have significant concave parts, T_K presents plateaus (see Fig. 2 for an example). (Witkin, 1983) suggested that the features of a signal that tended to be prominent were such features that showed stability over scale. Then, in line with the ideas of Witkin, a stable scale is a plateau of the edge texture indicator.

Note that the edge texture indicator can have several plateaus (see Fig. 2). Consequently we use the concept of thickening of the mathematical morphology (Serra, 1982, 1988) to choose a scale among these plateaus.
With a disk B as structuring element, the thickening of a contour C by B generates a strip ΔC whose width directly depends on the radius of B (Fig. 3).

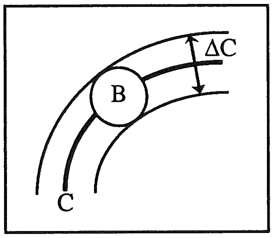


Fig. 3. Thickening of a contour C by a disk B.

The choice of an appropriate scale n can be then realized in two ways :

- the size of the structuring element B is unknown ; the scale n is given by the first plateau of the edge texture indicator. Such a choice allows to see the finest significant details of the discrete contour and allows to emphasize at best its concave parts.
- a size has been assigned to the structuring element B ; the maximal scale (for a minimal abstract in number of points) contained in a plateau is selected for the smoothing not to "overflow" the strip ΔC.

ABSTRACT OF A DISCRETE CONTOUR

The abstract of a discrete contour is made up of inflexion points, dominant points and tangent directions at these points. In this section we shall see how to derive these elements from a digital contour and we shall also see that the result of this operation is scale dependent. It stress the importance of the selection of an appropriate scale of study as a previous step.
The location of the convex and concave parts at a selected scale n is linked to the study of the points of C where the sign of A_n changes. This change of sign appears between two consecutive points p_i and p_{i+1} where angular sectors (p_{i-n}, p_i, p_{i+n}) and $(p_{i+1-n}, p_{i+1}, p_{i+1+n})$ are respectively positive and negative for example. These points are called *transition points* of the discrete contour C. Transition points p_i and p_{i+1} are not isolated if the arc $\{p_{i-n},... p_i, p_{i+1},... p_{i+1+n}\}$ contains other transition points. The arc regrouping the non isolated transition points corresponds to an arc where the convex and concave parts are in order rapidly ; this arc can then globally be seen at scale n as a rectilinear part of C. The two extremities of this arc are the points p_{j-n} and p_{k+n} (k>j) such that p_j and p_k be transition points and $\{p_{j-n},... p_j\}$ and $\{p_k,... p_{k+n}\}$ both contain any other transition point. The arc $\{p_{j-n},... p_{k+n}\}$ is a *transition part* of the discrete contour. This definition holds if p_i and p_{i+1} are isolated i.e. if p_i and p_{i+1} are the only two transition points of $\{p_{i-n},... p_i, p_{i+1},... p_{i+1+n}\}$.
Hence *inflexion points* are located in transition parts. The simplest way to estimate the position of an inflexion point from a discrete contour is to use the coordinates of the midpoint of the corresponding transition part. The direction of the straight line $(p_{j-n/2}, p_{k+n/2})$ gives a good approximation of the direction of the tangent at the inflexion point contained in $\{p_{j-n},... p_{k+n}\}$.
It has been suggested from the viewpoint of the human visual system (Attneave, 1954) that high curvature points or *dominant points* along a digital curve are rich in information content.

Unfortunately, direct measures of curvature cannot be applied to discrete contours because of the noise. It is thus essential to make indirect but accurate measures of curvature.
To any arc $\{p_i, \dots p_{i+n}\}$ of a discrete contour C can be superimposed the corresponding arc xy of the original continuous shape ($x \equiv p_i$ and $y \equiv p_{i+n}$). When assuming that the arc is regular enough the theorem of Rolle affirms that on the arc xy there is a point where the direction of the tangent to the curve is the direction of the segment [x, y]. Therefore, without knowing the reference shape, it is possible to have a sampling of tangent directions (succession of chords associated to the contour). The direction of the segment $[p_i, p_{i+n}]$ gives an estimation of a tangent direction. The application point of this tangent is the point of $\{p_i, \dots p_{i+n}\}$ which is the most far off the segment $[p_i, p_{i+n}]$. The points of application of the tangents are the *dominant points* of the discrete contour.
Given a scale n, the construction of the abstract of a discrete contour can be described in the following steps.

1) Delimit convex and concave parts, i.e., locate the transition parts of the contour.
2) Find the positions of the inflexion points and estimate the directions of their tangents.
3) Find the positions of the dominant points of the convex and concave parts of the contour. The tangent directions at these points are estimated at the same time.

SMOOTHING

The smoothing we are now going to present is computed starting from the abstract of a discrete contour and uses the theory of the envelopes of a one-parameter family of straight lines. This theory can be briefly summarized by the following statement :
in plane geometry, the straight line family $\{D_t\}_{t \in I}$ whose general equation is given by

$$\alpha(t)\cdot x + \beta(t)\cdot y + \gamma(t) = 0 \qquad \text{with} \quad \alpha, \beta \text{ and } \gamma \in C^2(I, \mathbf{R})$$

in general admits an envelope generated by the point common to D_t and to the straight line $D_{t'}$ when t varies

$$\alpha'(t)\cdot x + \beta'(t)\cdot y + \gamma'(t) = 0$$

whose equation is obtained when cancelling the derivative in respect to t of the first member of the equation of D_t.

The reader will find in the literature, for example in (Lelond-Ferrand, 1963), complementary details about this subject.
Let A and B be two distinct points of the plane and let T_A and T_B be two non collinear vectors at these points. Assume that the straight line (A, B) is parallel neither to T_A nor to T_B. Denote by Q the intersection point of the straight lines having respectively the same direction as T_A and T_B and denote by θ the angle formed by [Q, A] and [Q, B]. Finally, set a = QA, b = QB and define an orthonormal reference such that A(a, 0) and B(b.cosθ, b.sinθ).

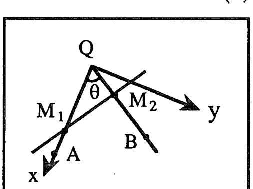


Fig. 4. Notations used in the smoothing step.

We study the family of straight lines which meet (A, Q) and (B, Q) respectively in A and B. A straight line of the family is, for example, the straight line (M_1, M_2).
Consider a function $t \rightarrow l(t)$ from an interval I of **R** into **R** such that $l(a) = 0$ and $l(0) = b$. It means that the distance $M_1Q + QM_2$ is equal to $t + l(t)$ between the points $M_1(t, 0)$ of (A, Q) and $M_2(l(t)\cdot\cos\theta, l(t)\cdot\sin\theta)$ of (B, Q). Thus, the function l(t) can be expressed by

$$l(t) = \frac{-b}{a}(t - a) \tag{1}$$

The equation of the straight line (M_1, M_2) is immediately deduced

$$(l(t)\cdot\sin\theta)\cdot x + (t - l(t)\cdot\cos\theta)\cdot y = t\cdot l(t)\cdot\sin\theta \tag{2}$$

To get the envelope, restrict the interval I = [0, a] and refer to the recall made in the beginning of this section ; it finally comes

$$f(t) = \begin{cases} x = \dfrac{1}{a}\left[t^2 + \dfrac{b(t-a)^2}{a}\cos\theta\right] \\ y = \dfrac{b(t-a)^2}{a^2}\sin\theta \end{cases} \tag{3}$$

Let $S = \{s_j\}$, j being a positive integer, be the set of points of the abstract of a discrete contour C. According to the principle construction of such an abstract, for any pair (s_j, s_{j+1}) belonging either to a convex or to a concave part of C, the intersection of the tangents applied in s_j and s_{j+1} exists. Thanks to Eq. 3 it is then possible to compute an envelope starting from the pair (s_j, s_{j+1}).
However, any pair of points (s_j, s_{j+1}) can generate a straight line parallel to one of the two tangents. Consequently, the straight line family $\{D_t\}_{t\in I}$ cannot be correctly defined and it is impossible to compute an envelope starting from such a pair of points. Actually, in such a case, it is useless to try to compute an envelope : a significant rectilinear part (e.g. a flat part) of C appears ; s_j and s_{j+1} are the extremities.
The same comment would apply if two successive tangents are collinear. This configuration can only appear when at least one point of (s_j, s_{j+1}) belongs to a transition part i.e. at least one point of (s_j, s_{j+1}) is an inflexion point.
The set of envelopes and possible segments forms the smoothing of a discrete contour. The whole smoothing method can in short be described as follows.

1) Compute the edge texture indicator of the digital contour C under study.
2) Thanks to the edge texture indicator select a scale of study.
3) With the scale selected in 2), construct the abstract of the discrete contour C.
4) Starting from the abstract constructed in 3), compute the smoothing.
5) If a size has been assigned to the structuring element B, test if the smoothing is contained in the strip generated by the thickening of C by B.
6) If 5) fails, go to 2) to select another scale.

In the previous lines we exposed the theorical principle of the adaptive discrete contour smoothing. We are now going to see results obtained on some typical examples.

RESULTS

Figure 2 shows a discrete contour with its edge texture indicator. In that case we can notice that the edge texture indicator is a staircase function. The plateaus correspond to the scales for which significant convex and concave parts are detected. The edge texture indicator of the contour of a digitalized convex object has only one plateau : the plateau where the number of sign changes becomes null and remains null. It is illustrated by the triangle of Figure 5.

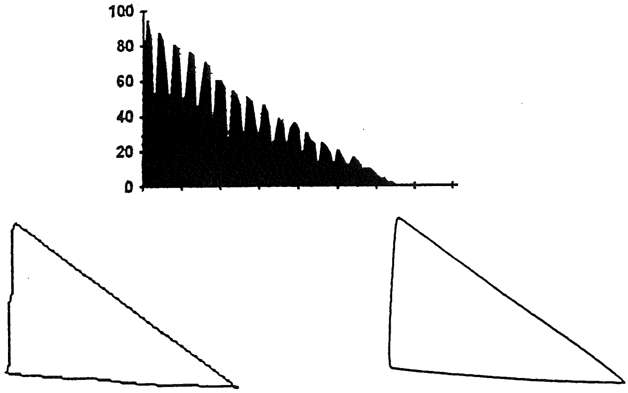


Fig. 5. A noisy contour of a triangle with its edge texture indicator and its smoothing.

Figure 6 shows the distribution of the points of the abstract of a discrete contour. We clearly see that the parts having an important curvature are much better represented than the approximately rectilinear parts. The smoothing is automatically adapted to the aspect of the contour under study. It does not depend on a threshold whose value would be to determine more or less empirically.

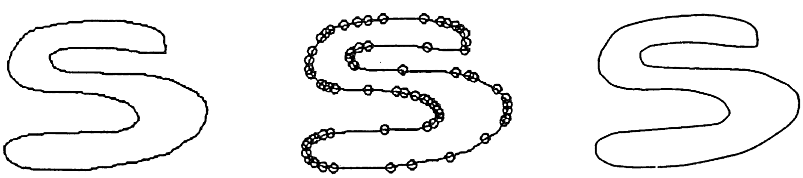
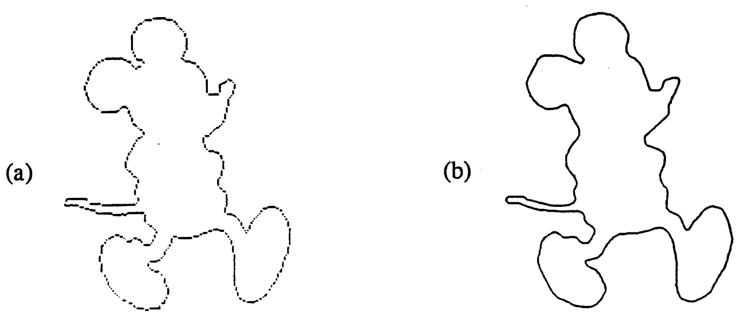
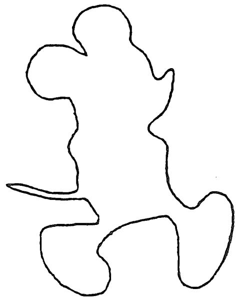


Fig. 6. A discrete contour with the points of its abstract and the final result of the smoothing obtained without any entry parameter.

All the smoothings previously presented have been obtained without any entry parameter. Figure 7 provides an example of smoothings computed at different scales. The discrete contour to study is the contour of the shadow of the famous Mickey Mouse. Its first smoothing (Fig. 7(b)) did not require any entry parameter. Given 2 and 6 as entry parameters, smoothings of Figures 7(c) and 7(d) are respectively obtained.



(c)



(d)

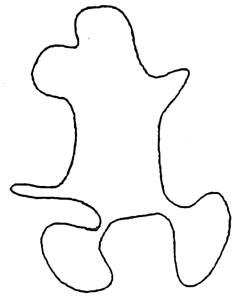


Fig. 7. (a) The contour of the digitalized shadow of Mickey Mouse. (b) The smoothing obtained without any entry parameter. (c) The smoothing computed when a 2 pixels parameter has been given. (d) A 6 pixels parameter has been provided.

The most global information remains obvious when the scale becomes large. As the quality of a smoothing depends on its application, the degree of liberty left for the scale is then very interesting.

CONCLUSION

In this paper, we dealt with an original discrete contour smoothing. It allows to recover the underlying continuous feature of a discrete contour seen at a determined scale. The edge texture indicator provides a scale adapted to each contour. An optionnal entry parameter (the radius of the disk in the thickening step) can be given by an operator or by another computing level for a more precise scale selection. This parameter has an intuitive character since it represents the maximal gap (in number of pixels) between the discrete contour under study and its smoothing.
The smoothing method is based on the theory of the envelopes of a one-parameter family of straight lines. It is computed starting from the abstract derived from the digital contour to study. In such an abstract, the parts of the discrete contour with significant curvature are more accurately represented than the parts of lesser curvature. In other words the parts with bends are described with more points than the others. Finally the complete smoothing method does not require any experimental threshold.

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