

**PROBABILISTIC ASPECTS OF EVALUATION OF FIBRE-MATRIX ADHESION FROM PULL-OUT FIBRE LENGTHS ON FRACTURE SURFACES OF SHORT FIBRE COMPOSITES.**

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**ABSTRACT**

A new method of evaluation of fibre-matrix adhesion in short fibre composites is proposed. The method is based on a comparison of experimental and model probability distribution functions of fibre pull-out length. The real pull-out lengths are measured on scanning electron micrographs of composite fracture surfaces. Two models for prediction of pull-out lengths are based on a known fibre length distribution in injection-moulded composites.

**Keywords:** short fibre composite, fibre-matrix adhesion, probability distribution function, fibre pull-out length.

**INTRODUCTION**

It is well known that the level of adhesion between fibres and matrix affects the ultimate thermomechanical properties of a composite. The usual experimental methods for estimation of adhesion are the fibre pull-out method and the fragmentation test, both relied on the use of single and long fibre. The most common techniques have been reviewed by Herrera-Franco and Drzal (1992). An indirect measure of the strength of an interfacial bond in laminates with continuous aligned fibres can be obtained from the fracture surface. When unidirectional composite is tested in longitudinal tension, an extensive fibre pull-out appears in case of weak interfacial adhesion. Calculation of the interfacial shear strength in composite with elastic fibre and plastic matrix have been carried out in the literature by Kelly-Tyson (1965), expression based on the assumption of the maximum fibre pull-out length on fracture area being equal to half the critical fibre length. Random orientation and various length distribution of short fibres in injection molded composites are a fundamental obstacle to using this simple method. No satisfactory method of measuring of the bond level in short fibre composite with higher fibre volume fraction has been described so far.

### MATERIALS AND EXPERIMENTAL METHODS

The principal materials used in our investigation are listed in Table 1. Injection-moulded specimens for a Charpy impact tester were made from commercially available polymers and glass fibres.

Table 1. List of composite materials with 30 wt.% glass fibres.

Matrix	Designation
Polypropylene (homopolymer)	Taboren 30GE
Polypropylene (homopolymer)	Mosten 30GE (Union Carbide agent)
Polypropylene (modified PP)	Propathene HW60GR30 (ICI commercial product)
Polyamid 6 (Spolana)	Spolamid 30GE
Polyamid 6 (Rhône-Poulenc)	Technyl 30GE

Microstructural parameters of the composite such as fibre diameter and orientation distribution were determined by a measurement on optical micrographs of polished sections cut from the specimens. The fibre length distribution  $F$  was evaluated by a direct measurement of the length of each fibre after separation of fibres from under-surface layers of the composite after burning off the matrix.

Very sharp and shallow notches were made in a width direction of the centre span of Charpy specimens (cross-section 4x10 mm, 55 mm long) as initiators of brittle crack. The fracture surfaces were examined with a scanning electron microscope (SEM). Fibre pull-out length distribution  $F_e$  was determined from micrographs of the under-surface regions with unidirectionally aligned fibres near the notch tip (see Fig.1). Inclination angle  $45^\circ$  was taken into account in the true pull-out length calculation.

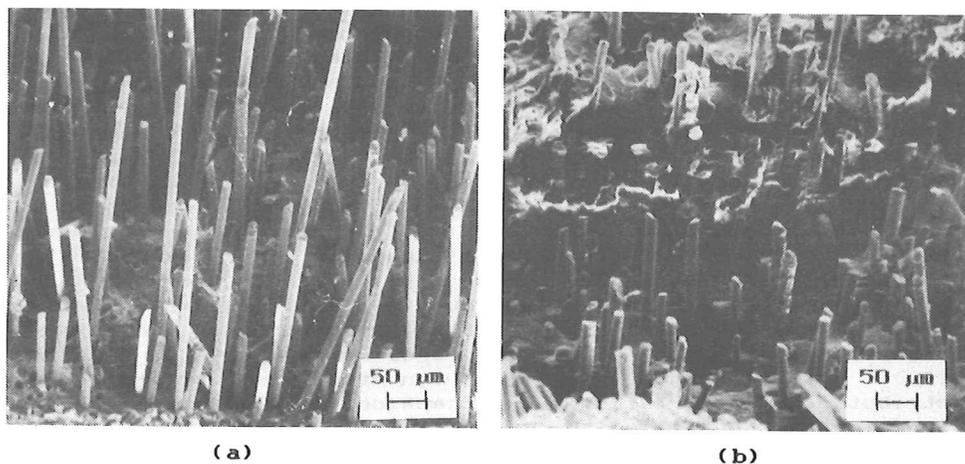


Fig.1. Scanning electron micrographs of fractured specimens of injection moulded glass fibre reinforced plastics: (a) polypropylene matrix, b) polyamide matrix (tilt angle is  $45^\circ$ )

## THEORETICAL MODELS

Our theoretical model of spatial distribution of parallel fibres in the matrix assumes Boolean segment process, i.e. centres of segments form a Poisson point process. The fibre lengths  $L$  are well described by a Weibull distribution

$$F(L) = 1 - \exp \left[ - \left( \frac{L}{\theta} \right)^\beta \right], \quad L \geq 0 \quad (1)$$

Let us now consider the situation when a brittle transverse crack of the matrix propagates across the composite and no fibre failure occurs. The result is represented by model A based on the assumption that fibres are only pulled out on their shorter side (see Fig.2, model A). Then the theoretical distribution function  $G$  of the pull-out length of fibres  $l$  is

$$G(l) = F(2l) + 2l \int_{2l}^{\infty} \frac{F(L)}{L} dL, \quad l \geq 0 \quad (2)$$

where the integral part defines the incomplete gamma function. However, for commercially available compounds with low fibre length and a strong fibre-matrix bond the model has to be modified to include:

- partial sliding fibres in matrix at their longer part
- tension failure of fibres

We propose a model B for partial slide  $l_v$  of the fibre length  $l_p$  (see Fig.2, model B)

$$l_v = l_p \exp[-(L-l_p)/c], \quad c > 0 \quad (3)$$

where  $l_p$  is pull-out length in model A. (3) gives the protruding length of fibre  $l_t$

$$l_t = l_p + l_v = l_p (1 + \exp[-(L-l_p)/c]) \quad (4)$$

Because there is a positive failure probability  $P$  of the longest fibre bridging the brittle crack of matrix (see Fig.2), we assume (for unknown critical length of fibres) that all fibres longer than the quantile  $L_{0,95}$  of original length  $L$  have a failure probability  $P = 1$ . There exists a constant  $K$  such that  $P = 0$  for  $L \leq K$  and  $P = (L-K)/(L_{0,95}-K)$  for  $K \leq L \leq L_{0,95}$ . The random position  $B$  of fibre failure is modelled by a fixed distribution function  $H$ :

$$H(x) = \frac{x^2}{L l_p}, \quad x \in \langle 0, l_p \rangle \quad (5)$$

$$H(x) = 1 - \frac{(L-x)^2}{L(L-l_p)}, \quad x \in \langle l_p, L \rangle \quad (5)$$

with mode  $l_p$  corresponding to maximal stress location. Assuming

not more then one failure of the fibre we obtain the total length of protruding

$$\begin{aligned}
 l_t &= (l_p - B) (1 + \exp [-(1-l_p)/c]) , & B < l_p \\
 &= (B - l_p) (1 + \exp [-l_p/c]) , & l_p \leq B < 2l_p \\
 &= l_p (1 + \exp [-(B - l_p)/c]) , & B \leq 2l_p
 \end{aligned}
 \tag{6}$$

The formula for the distribution function of  $l_t$  given by (4) and (6) is analytically insolvable. However, a Monte-Carlo algorithm using (1) - (6) enables to obtain the value of  $l_t$ . Thus for given  $c$ ,  $K$  and parameters  $\theta$ ,  $\beta$  of the random length  $L$  in (1), an empirical distribution function  $F_s$  of  $l_t$  is obtained by computer simulation. The desired optimal values  $c$ ,  $K$  are solutions of the problem:

$$\min_{c, K} Q, \text{ where } Q = \max_l |F_\theta(1) - F_s(1)| \tag{7}$$

$F_e$  being the experimental distribution function of pull-out length in the specimen.

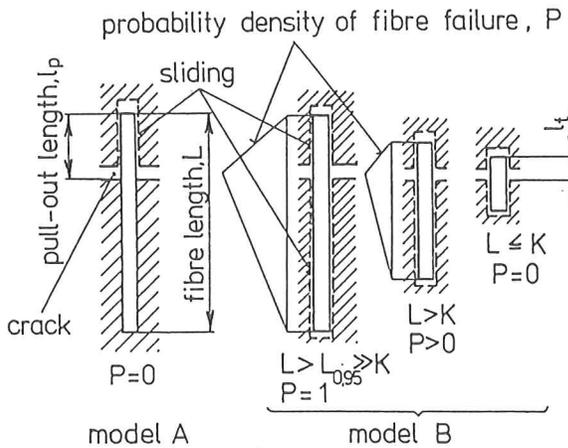


Fig.2. Schematical diagram of short fibre composite fracture ahead of a notch. Model A - without fibre failure and one-side sliding of fibre, model B - with fibre failure and two-side sliding.

RESULTS AND DISCUSSION

The curves of theoretical probability distribution function  $G$  (model A) and experimental distribution function (cumulative frequency)  $F_e$  of pull-out lengths, original length distribution  $F$ , and computer simulated distribution  $F_s$  (model B) for two composite materials, Taboren 30GF and Spolanid 30GF, are shown in Fig.3.

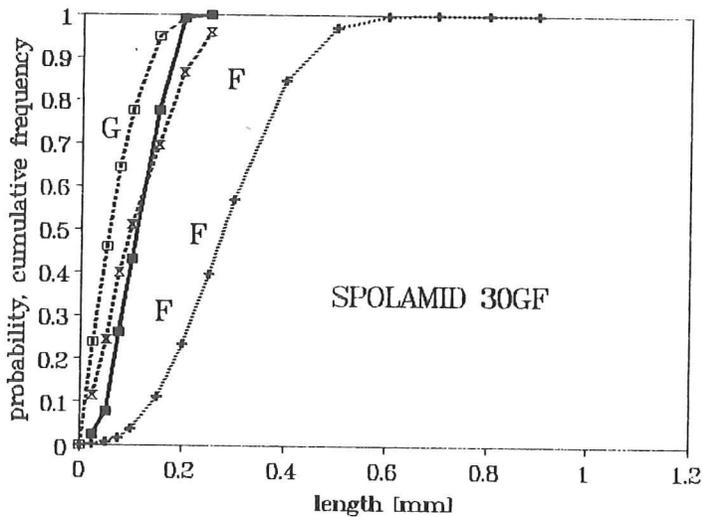
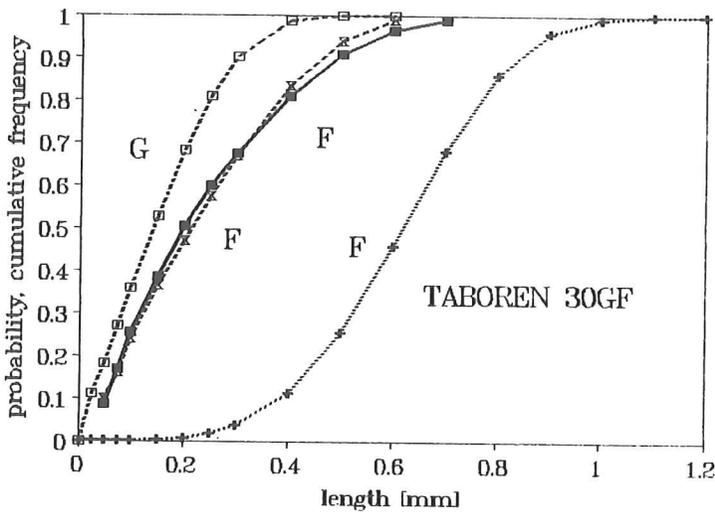


Fig.3. Probability distribution of length F, experimental pull-out length  $F_e$ , theoretical pull-out length G (model A) and computer simulated distribution  $F_s$  (model B).

When either the distribution function G is on the right side of the experimentally determined function  $F_e$  or there is an intersecting of G and  $F_e$  and a maximum fibre pull-out length

$l_{max}$  of the distribution  $F_e$  is smaller  $Lo_{,95}/2$  ( the theoretical maximum pull-out length), a significant fibre failure occurred during crack propagation. The fibre-matrix bond strength can be calculated from a critical fibre length  $l_c = 2l_{max}$  by the Kelly-Tyson expression.

The polypropylene composite (Fig.3 above) exhibits fibre length  $Lo_{,95} = 0,887$  mm and maximum pull-out length  $l_{max} = 0,55$  mm. The critical length  $l_c$  can not be evaluated. However, there is intersectioning of the distribution  $G$  and  $F_e$  for a polyamide composite and maximum experimental pull-out length of fibre  $l_{max} = 0,18$  mm is lower the theoretical maximum pull-out length  $Lo_{,95}/2 = 0,235$  mm ( $Lo_{,95} = 0,47$  mm), see Fig.3 below. This is a confirmation of the failure of the longest fibres. Therefore, the critical length of fibre can be obtained as  $l_c = 2l_{max} = 0,36$  mm. The other  $l_c$  have been treated identically (see column  $l_c$  in Table 2).

Results of the computer simulation are shown in Table 2.

Table 2. Variation in model parameters with composite type.

short fibre composite (abbr.)	parameters						
	$\theta$	$\beta$	$Lo_{,95}$ mm	K	c	$\frac{K}{Lo_{,95}}$	$l_c$ mm
MOSTEN	0,578	2,31	0,93	0,315	0,15	0,339	0,8
TABOREN	0,676	4,05	0,887	0,865	0,78	0,975	
PROPATHENE	0,663	3,467	0,909	0,595	0,09	0,654	0,62
SPOLAMID	0,319	2,806	0,47	0,405	0,56	0,861	0,36
TECHNYL	0,295	2,412	0,465	0,334	0,46	0,718	0,32

It may be concluded that pull-out of fibres becomes negligible for lower value of parameter c and no major fibre fracture occurred during crack propagation for values of normalized parameter  $K/Lo_{,95}$  near to 1.  $F_g$  gives a better fit of the  $F_e$  for material in which prevailing pull-out fibres occurs, it follows that the model B is appropriate. It is the objective of further study to obtain a better fit for materials with extensive failure of fibres. This can be solved by increasing the number of parameters in a model of probability and location of fibre failure.

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#### REFERENCES

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