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THE ESTIMATION OF ERRORS IN EVALUATING SPHERE SIZE DISTRIBUTION DUE TO VARIATIONS IN SECTION THICKNESS

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ABSTRACT

A simple expression is presented for the estimation of the errors in unfolding sphere size distributions and population densities incurred by deviations in the actual section thickness from its putative value. The error factor in determining population density for given actual and putative section thicknesses (Rose,1982) is expressed as a scalar multiple of a matrix for unfolding population densities and may readily be used in conjunction with the tables of these matrices previously published (Rose, 1980). It is shown that if the probability distribution of section thicknesses in the sampling fields is known, an estimate of the gross error in estimating population density incurred by variations in section thickness in the sampling fields may be made. It is further shown that in general the overall error in estimating population density is zero only if the probability distribution of section thicknesses is symmetrical about the putative value.

INTRODUCTION

The problem of estimating the errors in evaluating the size distribution and population density of a population of opaque spheres randomly dispersed throughout a transparent phase, from data obtained by transmission microscopy, incurred by deviations in the section thickness from its putative value has been discussed (Rose, 1982) and a general solution using matrix algebra suggested.

In the present communication it is shown that the matrix equation for the magnitude of error in estimating the numerical density of a population of spheres of specified diameter may be simplified to a form which is readily usable in conjunction with the tables for unfolding sphere population densities previously published (Rose, 1980).

THEORETICAL TREATMENT

Using the theoretical treatment and notation presented in previous works (Rose, 1980; 1982), it can be shown that the factor of error which is incurred in the estimation of the population density of spheres randomly dispersed in a transparent phase by a deviation of the section thickness from its putative value is given by the equation (Rose, 1982):-

$$E(N_m) = Q_{m(p)}\, R_{km(a)} - 1 \tag{1}$$

Now, for sections of any thickness, T, the matrix R_{km} may be written (Rose, 1980),

$$R_{km(T)} = R_{km(T=0)} + \frac{T}{\Delta}\,\delta_{km}, \tag{2}$$

and hence,

$$R_{km(a)} = R_{km(p)} + \frac{T_a - T_p}{\Delta}\,\delta_{km}. \tag{3}$$

Substituting into equation (1) gives,

$$E(N_m) = Q_{m(p)}\left\{R_{km(p)} + E(T)\,\frac{T_p}{\Delta}\,\delta_{km}\right\} - 1 \tag{4}$$

which may be rewritten as

$$E(N_m) = \sum_{j=1}^{J} P_{jk(p)}\left\{R_{km(p)} + E(T)\,\frac{T_p}{\Delta}\,\delta_{km}\right\} - 1 \tag{5}$$

which, since $P_{sr(T)} = R^{-1}_{rs(T)}$ (Rose, 1980), simplifies to

$$E(N_m) = \sum_{j=1}^{J}\delta_{jm} + \sum_{j=1}^{J} E(T)\,\frac{T_p}{\Delta}\,P_{jm(p)} - 1$$

$$= E(T)\,\frac{T_p}{\Delta}\,Q_m(p). \tag{6}$$

Since in practice the size distribution and population densities are evaluated on the basis of the putative section thickness, equation (6) provides a particularly simple means by which the error factors of population density associated with a specific error factor of section thickness may be estimated in practice.

DETERMINATION OF THE ERROR IN ESTIMATING POPULATION DENSITY FROM THE PROBABILITY DISTRIBUTION OF SECTION THICKNESSES

Let $Pr(T)dT$ denote the probability that T_a lies between T and $(T+dT)$.

The error in estimating N_m, weighted for variations in section thickness in the sampling field, is then

$$E_w(N_m) = \int_0^\infty Pr(T)E(N_m)_T dT, \tag{7}$$

which substituting equation (6) for $E(N_m)_T$ gives,

$$E_w(N_m) = \frac{T_p}{\Delta} Q_{m(p)} \int_0^\infty Pr(T)E(T)dt. \tag{8}$$

It follows that in general $E_w(N_m)$ is zero only if

$$\int_0^\infty Pr(T)E(T)dT = 0. \tag{9}$$

Since $E(T)$ is anti-symmetric with respect to T about T_p, it follows that equation (9) is, in general, satisfied only when $Pr(T)$ is symmetric about T_p.

REFERENCES

Rose PE. Improved tables for the evaluation of sphere size distributions including the effect of section thickness. J Microsc 1980;118:135-141.

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