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STEREOLOGICAL ANALYSIS OF FRACTURE ROUGHNESS PARAMETERS

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ABSTRACT

A procedure for obtaining the area of irregular fracture surfaces in terms of profile (R_L) and surface (R_S) roughness parameters is presented. A parametric equation for accomplishing this objective is derived and compared to others purporting to do the same. The analytical results are evaluated with all known experimental data and good agreement is obtained with the equation

$$R_S = (4/\pi)(R_L-1) + 1.$$

Key words: Nonplanar surfaces, parametric equations, quantitative fractography, roughness parameters, vertical sections.

INTRODUCTION

In quantitative fractographic studies, we are severely limited by the extent of angular sampling that is possible over the fracture surface. Tilting in the SEM stage quickly produces unrecognizable topography and serious overlapping, while stereoimaging is useful only with fairly flat facets. Sampling by section planes through the fracture surface automatically limits the possible angles of subsequent sections. Consequently, most sampling of fracture surfaces by sectioning is done with "vertical" sections (perpendicular to the effective plane through the fracture surface.)

Generally we have a partially-oriented fracture surface rather than one with randomly-oriented surface elements. Vertical sections then produce a fracture profile whieh is also partially-oriented. For this reason, we would like to have a relationship between surface roughness and profile roughness that takes partial orientation into account. An equation that accomplishes this objective, and overcomes some limitations of other expressions, is discussed below.

BACKGROUND

Early attempts to express the "roughness" of a nonplanar fracture surface and its profiles produced several kinds of roughness parameters (El-Soudani, 1978; Gray, et al., 1983; Hsu, 1962; Pickens and Gurland, 1976; Shieh, 1974; see Underwood, 1984; Underwood and Chakrabortty, 1981; Ward, 1975; Wright and Karlsson, 1983). Lately, the overwhelming consensus has centered on two simple, physically meaningful roughness parameters: R_S and R_L (Coster and Chermant, 1983; Exner and Fripan, 1985; Underwood, 1986b; Wright and Karlsson, 1983). R_S, the surface roughness parameter, is defined by

$$R_S = S/A' \quad (1)$$

where S is the actual area of the fracture surface and A' is its projected area. R_L, the profile roughness parameter, is defined by

$$R_L = L/L' \quad (2)$$

where L is the measured line length and L' is its projected length. Both the profile projection line and the fracture surface projection plane are usually selected to lie parallel to some arbitrary "effective" fracture plane. Both R_S and R_L are dimensionless ratios and depend only on the magnitude of the surface area or line length, respectively. For flat planes or straight line traces parallel to their projections, both R_S and R_L equal unity.

These parameters have limits of 1 and ∞. The minimum value is represented by a completely oriented surface lying parallel to its projection plane, while a surface with infinitely large area represents the maximum value. Fracture surfaces with configurations between the two extremes are called "partially oriented" (Saltykov, 1974; Underwood, 1970). Neither R_S nor R_L depend on the angular orientation of their elements. Thus, a "random" configuration* is undefined by the roughness parameters and cannot be used as a roughness "limit." The differences hinge on the type of sampling employed - - for the general stereological equations to apply, random angular and locational sampling must be employed. For the roughness parameters to be valid as defined, "directed" measurements are necessary and must be applied consistently throughout.

The actual area of an irregular fracture surface is difficult to obtain with any degree of precision. Thus, considerable effort has been expended lately in assessing the various ways by which this area can be determined. When vertical sections are cut through the fracture surface, profiles are generated whose geometrical characteristics are related probabilistically to those of the surface. Accordingly, R_S and R_L are intuitively felt to be related.

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*By a "random" configuration we refer to a uniform angular distribution of facet normals in three-dimensional sample space.

R_L is experimentally accessible and can be measured readily using automatic image analysis equipment (Underwood and Banerji, 1987). On the other hand, R_S, which is the quantity sought, must be calculated. Several parametric equations relating R_S to R_L have been offered in the literature (Coster and Chermant, 1983; El-Soudani, 1978; Underwood, 1986b; Wright and Karlsson, 1983). The discrepancies are not minor, and in some cases the assumptions embodied in the derivations are questionable. This paper attempts to clarify the basic differences between the various approaches, and to compare them with experimental data.

ANALYSIS

The two roughness parameters are related to the stereological quantities S_V, the surface area per unit volume, and L_A, the profile trace length per unit area. These two terms are connected by the general stereological equation (Underwood, 1970)

$$S_V = (4/\pi)\ L_A \qquad (3)$$

which is valid for any configuration of surface elements. If the surface elements are not oriented randomly, Eq. (3) is still applicable provided random sampling by section planes, test lines and/or measurement points is accomplished. If the surface elements are oriented randomly, then sampling can be performed in any preferred direction and Eq. (3) is still applicable.

For oriented or partially-oriented surfaces, special equations exist that provide additional information of a directional nature (Underwood, 1970). Eq. (3) is still valid, of course, but significant simplifications are achieved in some cases with directed measurements. For example, a special case of Eq. (3) applies to the completely-oriented surface, which represents a minimum area configuration. When section planes are cut perpendicular to the oriented surface, the traces are straight lines and

$$(S_V)_{or} = (L_A)_{or} \quad . \qquad (4)$$

This expression requires directed sectioning perpendicular to the oriented (planar) surface. A similar relationship applies to ruled surfaces (generated by the parallel translation of a straight line). The profile can have any degree of complexity, so the area of a ruled surface can be greater than that of the completely-oriented surface. The equation is

$$(S_V)_{ruled} = (L_A)_{\perp} \qquad (5)$$

where again the section planes must be directed perpendicular to the ruled surface elements.

A general equation that expresses the gamut of configurations between the completely-oriented and random fracture surfaces (Underwood and Banerji, 1983) is

$$S_V = K_{\Omega}\ L_A \qquad (6)$$

The two extremes are represented by Eqs. (4) and (3), thus the limits of the coefficient K_Ω are

$$1 \leq K_\Omega \leq \frac{4}{\pi} \tag{7}$$

K_Ω may be considered to be a function of the Degree of Orientation for lines in a plane, Ω_{12}, which is defined by

$$\Omega_{12} = L_{or}/L \tag{8}$$

where L_{or} is the length of the oriented components of a profile of total length L. Ω_{12} can vary between the limits of 0 and 1, where 0 represents no oriented components (a completely random line) and 1 means a completely oriented line. Since Ω_{12} is determined by directed measurements, $(P_L)_\perp$ and $(P_L)_\parallel$, then the use of K_Ω must also be restricted to directed measurements.

In order to evaluate K_Ω over the range of partially oriented lines between the two extremes set up above, we introduce the parameter R_f, defined by

$$R_f = \frac{L - L'}{L} \tag{9}$$

where L is the true profile length and L' is its projected length in the selected direction. It is apparent that

$$R_f = 1 - \frac{L'}{L} = 1 - \frac{1}{R_L} \tag{10}$$

where R_L is the roughness parameter defined above. The values of R_f vary between 0 (for the oriented case when L = L') and approach 1 (for the extremely complex trace where L $\gg$ L'), or

$$1 \geq R_f \geq 0 \quad . \tag{11}$$

We can now evaluate K_Ω in terms of R_f between the limits specified in Eqs. (7) and (11). Assuming linearity between the extreme configurations, we equate two values for the slope and obtain

$$\frac{K_\Omega - 1}{R_f} = \frac{4}{\pi} - 1 \quad . \tag{12}$$

Solving for K_Ω and substituting for R_f from Eq. (10) gives

$$K_\Omega = \left(\frac{R_L - 1}{R_L}\right)\left(\frac{4}{\pi} - 1\right) + 1 \quad . \tag{13}$$

This value of K_Ω can be substituted into Eq. (6).

However, we would rather express Eq. (6) in terms of roughness parameters. This can be accomplished readily as seen in Figure 1. The surface area per unit volume is

$$S_V = S/V_T = S/A'h = R_S/h \tag{14}$$

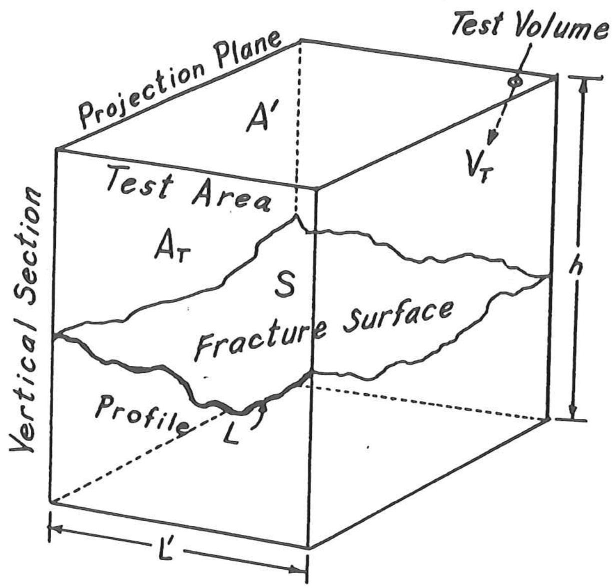

Figure 1. Arbitrary Test Volume Enclosing a Fracture Surface.

and the trace length per unit area is

$$L_A = L/A_T = L/L'h = R_L/h \tag{15}$$

Substituting for S_V and L_A in Eq. (3) yields

$$R_s = (4/\pi)\, R_L \tag{16}$$

and in Eq. (6) gives

$$R_S = K_\Omega\, R_L \tag{17}$$

Replacing K_Ω with the expression in Eq. (13) and simplifying, results in the useful equation

$$R_S = \frac{4}{\pi}\,(R_L - 1) + 1 \tag{18}$$

This parametric equation relates R_S linearly to the single parameter R_L for any degree of orientation between a completely oriented surface and one with a high degree of roughness. It is applicable between $R_L = 1$ and $R_L \gg 1$, which simplify to the completely-oriented case at the lower limit

$$(R_S)_o = (R_L)_o = 1 \tag{19}$$

and to Eq. (16) at the upper limit in R_L.

A two-parameter roughness equation has also been proposed (Underwood, 1986a). It involves both R_L and Ω_{12} explicitly and has the form

$$R_S = \left[\frac{4}{\pi} - \left(\frac{4}{\pi} - 1\right)\Omega_{12}\right] R_L \tag{20}$$

and is also based on directed measurements. For $\Omega_{12} = 1$ (completely-oriented profile), Eq. (19) is obtained, and for $\Omega_{12} \rightarrow 0$ (limiting case for an extremely complex configuration), we obtain Eq. (16).

Another parametric equation that has appeared from other sources (Coster and Chermant, 1983; Wright and Karlsson, 1983) is

$$R_S = \left(\frac{2}{\pi - 2}\right)(R_L - 1) + 1 \tag{21}$$

This relationship is based on limits of $1 \leq R_S \leq 2$ and $1 \leq R_L \leq \pi/2$. The upper limits derive from the general stereological equations (Underwood, 1970, Underwood, 1972) for mean projected area $\bar{A}'$ and mean projected length $\bar{L}'$, viz.

$$S = 2\,\bar{A}' \tag{22}$$

and

$$L = (\pi/2)\bar{L}' \tag{23}$$

for a surface of area S and a line in a plane of length L, respectively. Under these conditions we see that $R_S = 2$ and $R_L = \pi/2$. However, there is an important difference here for Eqs. (1) and (2). The latter definitions do not involve mean projected quantities, as do Eqs. (22) and (23). The results from Eqs. (22) and (23) are valid for all fracture surface configurations, if randomly sampled, and do not discriminate between completely-oriented, partially-oriented, or random surfaces. Moreover, finite values for the upper limits ($R_S = 2$ and $R_L = \pi/2$) are meaningless when applied to the roughness parameters as defined by Eqs. (1) and (2). There, the values of both R_S and R_L must approach infinity as the fracture surface becomes more and more complex. Thus, there is no physical justification for the cut-off at $R_S = 2$ $R_L = \pi/2$. These comments will be examined in the next section where the various parametric equations and actual experimental points are shown together on a plot of R_S vs R_L.

EXPERIMENTAL

The validity of parametric equations that relate R_S and R_L can be assessed with experimental data or with data obtained from a computer simulated fracture surface (CSFS) of known characteristics (Underwood and Underwood, 1982; Underwood and Banerji, 1983). R_L is measured directly from vertical sections through the fracture surface. However, R_S for real fracture surfaces must be calculated and can be determined in several ways: (1) by a modified form (Underwood and Banerji, 1987) of the Scriven and Williams' (Scriven and Williams, 1965) analysis based on the angular distribution of linear elements along the profile; (2) by parametric equations such as Eq. (18); and (3) by a triangulation method (Exner and Fripan, 1985; Wright and Karlsson, 1983) based on stereophotogrammetry of the fracture surface. Also, both R_L and R_S can be calculated readily for any degree of distortion of the CSFS. Methods (1) and (3) embody the assumption of flat, finite-sized facets, so the results appear to better advantage with heavily faceted fracture surfaces. Method (2) is independent of any shape assumptions, so is more general.

All known pairs of R_S and R_L values are plotted in Figure 2 with respect to four superimposed curves. The median curve is Eq. (18) and is shown as a heavy solid line. The two limit curves appear as dashed lines and are defined by Eq. (16) for the upper limit and by Eq. (5) for the lower limit. Eq. (21) is also included as the dotted curve, but is shown primarily for comparison purposes.

The experimental points in Figure 2 include roughness data for 4340 steels (Banerji, 1986; Underwood, 1986b), $A\ell$-4% Cu alloys in four heat treat conditions (Banerji and Underwood, 1985; Banerji, 1986), Ti-alloys with 24 and 28 %V (Underwood and Chakrabortty, 1981; Underwood, 1986b), and an $A\ell_2O_3$ plus 3% glass ceramic material (Exner and Fripan, 1985). The horizontal bars through the triangles represent the 95 percent confidence limits of R_L-values from six serial sections for each of the $A\ell$-4% Cu alloys.

In general, the points fall satisfactorily close to the median curve. Some points (+,x,Δ) tend to lie more toward the upper limit curve, and these originate from fracture surfaces that are heavily faceted. It would appear that the faceted configurations conform closer to the assumptions of the analytical methods than the complex, partially-oriented 4340 fracture surfaces (O,$\square$). Of course, <u>all</u> points would fall on the upper limit curve if the surfaces were (or could be) sampled randomly.

The location of all points in Figure 2 are dependent on the accuracy with which R_L has been determined, which in turn depends on the accuracy with which L, the true profile length, is determined. We know there is a fractal variation in the apparent length of an irregular curve depending on the size of the measuring unit, η, used to estimate the profile length (Paumgartner, et al., 1981). We have proposed a procedure that eliminates the fractal variation in apparent profile length as a function of η (Underwood and Banerji, 1986). Extrapolated values of R_L for $\eta \rightarrow 0$ represent the "true" values of R_L and are designated by $(R_L)_o$. The corresponding values for "true" fracture surface area are denoted by $(R_S)_o$. The six filled circles in Figure 2 show the locations of these asymptotic values of $(R_L)_o$ and $(R_S)_o$. They all fall closely around the heavy median line, which greatly enhances the credibility of Eq. (18).

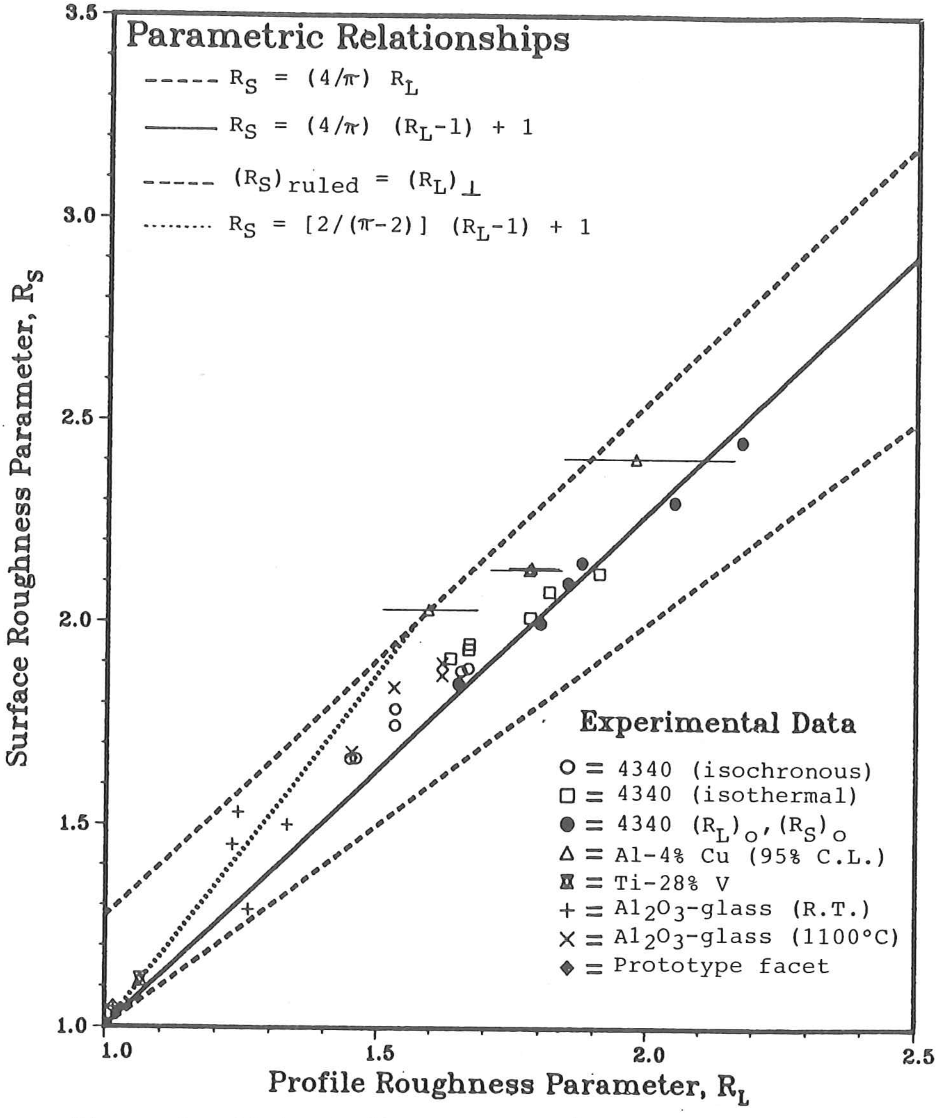

Figure 2. Plot of All Known Experimental Pairs of R_L, R_S.

CONCLUSIONS

Of the several parametric roughness equations available for calculating R_S from R_L, the most useful appears to be Eq. (18),

$$R_S = \frac{4}{\pi}(R_L - 1) + 1.$$

This relationship is based on directed, rather than random measurements. The important point emphasized here is that relationships based on general stereological equations are valid for any type of surface configuration, provided sampling is performed randomly. However, they give no information about the type of surface being investigated.

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